Sequence & Series

24 August 2021 05:07 PM

$a_n = n^2$ ← n^{th} term (some function of 'n')

Ex: $a_1 = 1^2 = 1$
$a_2 = 2^2 = 4$
$a_3 = 3^2 = 9$

Sum of n terms

$S_n = a_1 + a_2 + a_3 + \dots\dots + a_n$

$$\boxed{S_n = \sum_{r=1}^{n} a_r}$$

Ex 1: $a_n = 2n+5$, Find First 3 terms

$a_1 = 2(1)+5 = 7$

$a_2 = 2(2)+5 = 9$

$a_3 = 2(3)+5 = 11$

AP ... $\boxed{a=7,\ d=2}$

$a_n = an + b$ → AP

$a_n = a + (n-1)d$

$a + nd - d$

$a_n = a - d + nd$

$$\boxed{a_n = A + Bn}$$

Ex 3. $\boxed{a_1 = 1}$ $\boxed{a_n = a_{n-1} + 2}$

~~$a_n = a_{n-2}+7$~~

$n=2$: $a_2 = a_1 + 2$, $a_2 = 1+2$, $\boxed{a_2 = 3}$

$n=3$: $a_3 = a_2 + 2$, $a_3 = 3+2$, $\boxed{a_3 = 5}$

$n=4$: $a_4 = a_3 + 2$, $a_4 = 5+2$, $\boxed{a_4 = 7}$ ... $\boxed{a_5 = 9}$

Exercise 9.1

Q2- $a_n = \dfrac{n}{n+1}$

$a_1 = \dfrac{1}{1+1} = \dfrac{1}{2}$

$a_2 = \dfrac{2}{2+1} = \dfrac{2}{3}$

$a_3 = 3/4$

$a_4 = \dfrac{4}{5}$

$a_5 = \dfrac{5}{6}$

Q5- $a_n = (-1)^{n-1}(5)^{n+1}$

$a_1 = (-1)^0\, 5^2 = 25$

$a_2 = (-1)^1\, 5^3 = -125$

$a_3 = (-1)^2\, 5^4 = 625$

$a_4 = -3125$

$a_5 = +15{,}625$

Q 8 → $a_n = \frac{n^2}{2^n}$ $\qquad a_7 = \frac{7^2}{2^7} = \frac{49}{128}$

Q11 → $a_1 = 3 \qquad a_n = 3a_{n-1} + 2 \qquad \boxed{n>1}$

$a_2 = 3a_1 + 2 = 3(3)+2 = 11$ $\qquad a_3 = 3(11)+2 = 35$ $\qquad a_4 = 3(35)+2 = 107$

$a_5 = 3(107)+2 = 323$

Sequence $\quad 3, 11, 35, 107, 323 \ldots\ldots$

Q14 → Fibonacci... (Naturally occuring series) Youtube → Fibonacii, Golden Ratio } Disney

$a_1 = 1 \qquad a_2 = 1 \qquad a_n = a_{n-1} + a_{n-2} \qquad \boxed{n \geq 3}$

$a_3 = a_2 + a_1 = 1+1 = 2$ $\qquad a_4 = a_3 + a_2 = 2+1 = 3$ $\qquad a_5 = a_4 + a_3 = 3+2 = 5$

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\ldots$

Ex 4 In an A.P. if mth term is n and the nth term is m, where m ≠ n, find the pth term.

AP --- a, d → 2 variables

$a_m = n \qquad a + (m-1)d = n \quad -(1) \qquad a_p = a + (p-1)d$??

$a_m = n$ $\quad$ $a + (m-1)d = n \quad —(1)$ $\quad$ $a_p = a + (p-1)d$??

$a_n = m$ $\quad$ $a + (n-1)d = m \quad —(2)$

$md - \cancel{d} - nd + \cancel{d} = n - m \quad (1)-(2)$

$d\,\cancel{(m-n)} = -\cancel{(m-n)}$

$\boxed{d = -1} \quad —(3)$

from (1) & (3)

$a + (m-1)(-1) = n$

$a - m + 1 = n$

$\boxed{a = m + n - 1}$

p^{th} term $\quad a_p = a + (p-1)d$

$= (m + n - 1) + (p-1)(-1)$

$= m + n \cancel{-1} - p + \cancel{1}$

$\boxed{a_p = m + n - p}$

Ex6

$S_n = nP + \frac{1}{2} n(n-1) Q$ $\quad$ where P & Q are some constants ...

$S_1 = a_1 = 1(P) + \frac{1}{2}(1)(0)Q$

$\boxed{S_1 = a_1 = P} \quad —(1)$

$S_2 = a_1 + a_2 = 2P + \frac{1}{\cancel{2}}\cancel{2}(1)Q$

$\boxed{S_2 = a_1 + a_2 = 2P + Q} \quad —(2)$

$S_3 = a_1 + a_2 + a_3 = 3P + \frac{1}{\cancel{2}} \cdot 3 \cdot \cancel{2} \cdot Q = 3P + 3Q \quad —(3)$

from (1) & (2)

$P + a_2 = 2P + Q$

$\boxed{a_2 = P + Q}$

P

$P + (P + Q) + a_3$

$= 3P + 3Q$

$\boxed{a_3 = P + 2Q}$

$a_1, a_2, a_3,$ $\quad$ $\boxed{a = P}$

$a_1, a_2, a_3,$
$P, P+Q, P+2Q$

$$\boxed{a=P \quad d=Q}$$

Alternate.... (Method of differences)

$S_1 = P \Rightarrow \boxed{a_1 = S_1 = P}$

$$\boxed{d = a_2 - a_1 = Q}$$

$S_2 = 2P+Q \qquad a_2 = S_2 - S_1 = P+Q$

$d = a_3 - a_2 = Q$

$S_3 = 3P+3Q \qquad a_3 = S_3 - S_2 = P+2Q$

$d = a_4 - a_3 = Q$

$S_4 = 4P+6Q \qquad a_4 = S_4 - S_3 = P+3Q$

Example 6...

First term, Common difference..

$AP_1 = a_1 \,\&\, d_1$

$AP_2 = a_2 \,\&\, d_2$

$a_n = a+(n-1)d$

$$\boxed{S_n = \frac{n}{2}\left(2a+(n-1)d\right)}$$

$$\frac{S_n^1}{S_n^2} = \frac{3n+8}{7n+15}$$

$$\boxed{\frac{\cancel{\frac{n}{2}}(2a_1+(n-1)d_1)}{\cancel{\frac{n}{2}}(2a_2+(n-1)d_2)} = \frac{3n+8}{7n+15}}$$

$n=23$

To find Ratio of 12^{th} term...

$$\boxed{\frac{a_{12}^1}{a_{12}^2} = \frac{a_1+11d_1}{a_2+11d_2}}$$

$$\frac{2a_1+22d_1}{2a_2+22d_2} = \frac{3(23)+8}{7(23)+15}$$

$2a_2 + 2ad_2$ $\quad$ $7(23)+15$

$$\frac{a_1+11d_1}{a_2+11d_2} = \frac{77}{176} \cdot \frac{7}{16} \Rightarrow \boxed{a_{12}^1 : a_{12}^2 = 7:16}$$

$$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \end{array}$$

Extra

$$\frac{a_{17}^1}{a_{17}^2} = ??$$

$$\frac{a_1+16d_1}{a_2+16d_2}$$

We have

$$\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{3n+8}{7n+15}$$

Put... $n=33$

$$\frac{a_1+16d_1}{a_2+16d_2} = \frac{3(33)+8}{7(33)+15} = \frac{107}{246}$$

Ex8 Insert 6 number b/w 3 & 24 $\Rightarrow$ AP ... a, d

$3, \underline{A_1}, \underline{A_2}, \ldots\ldots, \underline{A_6}, 24$

$a_1 = 3$

$a_8 = 24$

$a_2 = A_1 = a+d = 3+3 = 6$

$a_3 = A_2 = a+2d = 3+2(3) = 9$

$\boxed{a=3}$ — (1)

$a+7d = 24$ — (2)

$7d = 21$ $\quad$ $\boxed{d=3}$

~~$a_4 = 12$~~ $\quad A_3 = 12$

~~$a_5 = 15$~~ $\quad A_4 = 15$

~~$a_6 = 18$~~ $\quad A_5 = 18$

$A_6 = 21$

Exercise 9.2

Q5 $\to$ $\quad a_p = \frac{1}{q}$

$a_q = \frac{1}{p}$

$AP \Rightarrow a, d$

$S_{pq} = \frac{1}{2}(pq+1)$

Given... $p \neq q$

$a+(p-1)d = \frac{1}{q}$ — (1)

$a+(q-1)d = \frac{1}{p}$ — (2)

from (3) & (1)

from (3) & (1)

$$a + (p-1)\frac{1}{pq} = \frac{1}{q}$$

$$a = \frac{1}{q} - \frac{(p-1)}{pq}$$

$$a = \frac{\cancel{p} - \cancel{p} + 1}{pq}$$

$$\boxed{a = \frac{1}{pq}} \quad -(4)$$

$$a + (q-1)d = \frac{1}{p} \quad (2)$$

$$pd - \cancel{d} - qd + \cancel{d} = \frac{1}{q} - \frac{1}{p}$$

$$d(\cancel{p-q}) = \frac{(\cancel{p-q})}{pq}$$

$$\boxed{d = \frac{1}{pq}} \quad -(3)$$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{pq} = \frac{pq}{2}\left(2\left(\frac{1}{pq}\right) + (pq-1)\frac{1}{pq}\right)$$

$$S_{pq} = \frac{pq}{2}\left(\frac{2}{pq} + 1 - \frac{1}{pq}\right)$$

$$S_{pq} = \frac{pq}{2}\left(\frac{1}{pq} + 1\right)$$

$$S_{pq} = \frac{1}{2}(1 + pq) = \underline{\frac{1}{2}(pq+1)}$$

Q6 → AP : 25, 22, 19, ... → $a = 25$, $d = -3$

$S_n = 116$

$n = ??$

$$\frac{n}{2}\left(2a + (n-1)d\right) = 116$$

$$\frac{n}{2}\left(2(25) + (n-1)(-3)\right) = 116$$

$$\frac{n}{2}\left(50 - 3n + 3\right) = 116$$

$$n(53 - 3n) = 232$$

$$53n - 3n^2 = 232$$

$$3n^2 - 53n + 232 = 0$$

$$n = \frac{-(-53) \pm \sqrt{(-53)^2 - 4(3)(232)}}{2(3)}$$

$$n = \frac{53 \pm \sqrt{2809 - 2784}}{6}$$

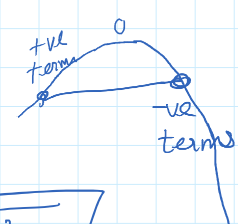
$$\boxed{n = 8}$$

Ex

4, 2, 0, -2, ...

$S = 4$

$S_4 = 4$



$$\boxed{n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\boxed{n = 8}$$

$$a_8 = a + 7d$$
$$= 25 + 7(-3)$$
$$= 25 - 21$$
$$= 4$$

$$n = \frac{53 \pm 5}{6}$$

$n = \frac{58}{6}$ or $\frac{48}{6}$

$n = 9.66$ ✗ / 8 ✓

~~$S_8 = 116$~~
~~$S_9 = 117$~~
~~$S_{10} = 115$~~

Q8 $\qquad S_n = pn + qn^2 \qquad d = ??$

M1

$$S_1 = p + q \implies a_1 = p + q$$
$$S_2 = 2p + 4q \qquad a_2 = S_2 - S_1 = p + 3q$$
$$S_3 = 3p + 9q \qquad a_3 = S_3 - S_2 = p + 5q$$
$$S_4 = 4p + 16q \qquad a_4 = S_4 - S_3 = p + 7q$$
$$S_5 = 5p + 25q \qquad \vdots$$

$$d_1 = a_2 - a_1 = 2q$$
$$d_2 = a_3 - a_2 = 2q$$
$$d_3 = a_4 - a_3 = 2q$$

$$\boxed{d = 2q}$$

Q10 ...

$AP \to a, d$

Sum of p terms of an AP $=$ Sum of q terms on an AP

$$S_p = S_q$$

$$\frac{p}{\cancel{2}}\left(2a + (p-1)d\right) = \frac{q}{\cancel{2}}\left(2a + (q-1)d\right)$$

$\begin{pmatrix} a > 0 & a < 0 \\ d < 0 & d > 0 \end{pmatrix}$

$$2ap + p(p-1)d = 2aq + q(q-1)d$$
$$2a(p-q) = (q^2 - q - p^2 + p)d$$
$$2a(p-q) = -(p^2 - q^2 + p - q)d$$
$$2a(p-q) = -((p-q)(p+q) + (p-q))d$$
$$2a\cancel{(p-q)} = -\cancel{(p-q)}\{p+q+1\}d$$

$S_{p+q} = ??$

$$\boxed{2a = -(p+q+1)d} \quad \text{—— (1)}$$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{p+q} = \frac{p+q}{2}\left\{-\cancel{\left(\frac{p+q}{}+1\right)}d + \cancel{(p+q-1)}d\right\}$$

$$\boxed{S_{p+q} = 0}$$

— x — x — x —

Q →

AP ⇒ A, D

$a_p = a$ ⟹ $A + (p-1)D = a$ — ①

$a_q = b$ ⟹ $A + (q-1)D = b$ — ②

$a_r = c$ ⟹ $A + (r-1)D = c$ — ③

③ − ①

$(r-p)D = c-a$

$(r-p) = \frac{c-a}{D}$ — ③'

$(p-q)D = a-b$

$p-q \Rightarrow \frac{a-b}{D}$ — ①'

$(q-r)D = b-c$

$q-r = \frac{b-c}{D}$ — ②'

Prove

$$\text{LHS} = \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

from ①, ② & ③

$$\left(\frac{A+(p-1)D}{p}\right)(q-r) + \left(\frac{A+(q-1)D}{q}\right)(r-p) + \left(\frac{A+(r-1)D}{r}\right)(p-q)$$

p, q, r

Alternate ... from ①' ②' & ③'

$$\frac{a}{p}\left(\frac{b-c}{D}\right) + \frac{b}{q}\left(\frac{c-a}{D}\right) + \frac{c}{r}\left(\frac{a-b}{D}\right)$$

a, b, c

① $A + (p-1)D = a$

$(p-1)D = a - A$

$(p-1) = \frac{a-A}{D}$

$$\boxed{p = \frac{a-A}{D} + 1}$$

— x — x — x —

Next class

Q15 →

$$AM = \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$$

for a & b

n = ??

$$AM = \frac{a+b}{2}$$

AM of a & b is $\frac{a+b}{}$

$AM = \frac{a+b}{2}$

AM of a & b is $\frac{a+b}{2}$

$\boxed{n=1}$

$$\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \frac{a+b}{2}$$

$a \neq b$

$$2a^n + 2b^n = (a+b)(a^{n-1} + b^{n-1})$$

$$2a^n + 2b^n = a^n + a^{n-1}b + b^{n-1}a + b^n$$

$$a^n + b^n = a^{n-1}b + b^{n-1}a$$

$$a^n - a^{n-1}b = b^{n-1}a - b^n$$

$$a^{n-1}\cancel{(a-b)} = b^{n-1}\cancel{(a-b)}$$

$$a^{n-1} = b^{n-1}$$

$$\left(\frac{a}{b}\right)^{n-1} = 1$$

$$\left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^{0}$$

$n-1=0$

$n=1$